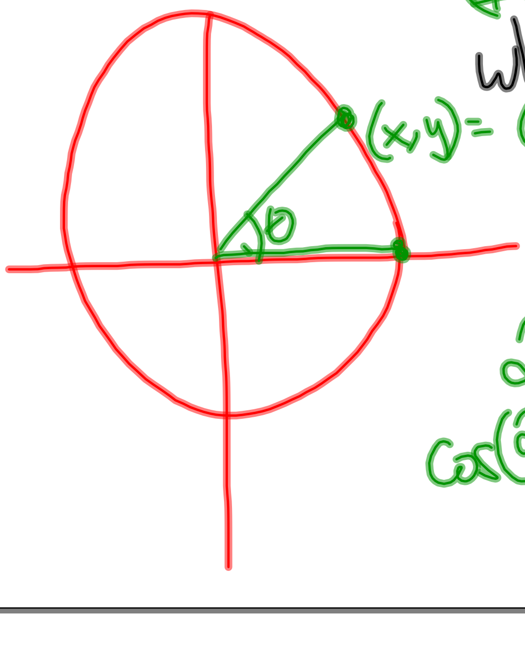
2014-09-03 day 6

24) If $\sin(a+b)=1$ and $\cos(a)=0$ then $\cos(2b)=?$

$$\sin(a+b)=\sin(a)\cos(b)+\cancel{\sin(b)\underbrace{\cos(a)}_{0}}$$

$$\sin(a)\cos(b)=1$$
$$(1)\quad(1)$$

what is b? if $\cos(b)=1$?




$(x,y)=(\cos\theta,\sin\theta)$

$\theta=0, 2\pi, 4\pi, \dots$

$2\theta=0, 4\pi, 8\pi, \dots$

$\cos(2b)=\cos(b)=1$

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Asymptotes - undefined places / vertical line on some calc.

Vertical asymptote is a place where the limit does not exist. (DNE)
[in fact the limit DNE, because it approaches infinity or negative infinity]

Holes do not effect the existence of a limit.
[A limit can exist at an x-value where the $f^n$ is undefined.

A limit IS a y-value.
[it is the y-value a $f^n$ "intends to reach" as x's get closer & closer to a value.

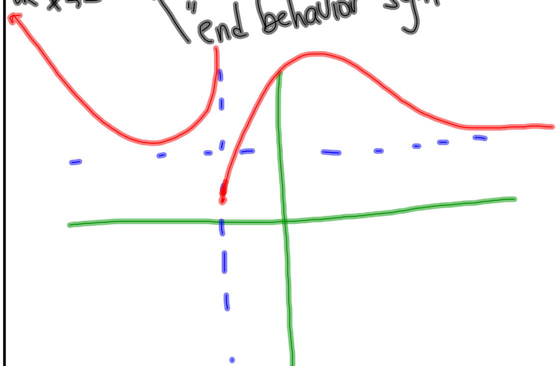
There are "one-sided" limits. We say left-side (or left handed) limit, and right-side/right handed

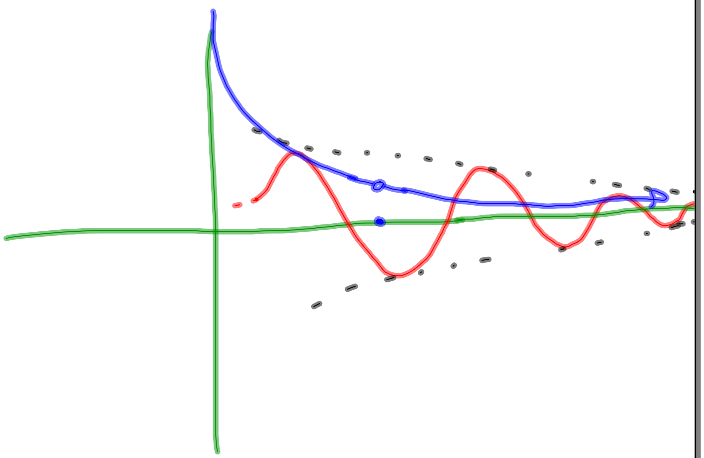
Two sided limits [or just "limit"] exist when both one-sided limits are equal.

$\lim\limits_{x\to a} f(x) = L$ / $\lim\limits_{x\to a^+} f(x) = L$ from the right / $\lim\limits_{x\to a^-} f(x) = L$ from the left

$\lim\limits_{x\to+\infty \text{ OR } x\to-\infty} f(x) = L$ means we have a horizontal asymptote. at $y=L$.

"end behavior" syntax





2014-09-03 day 6

Let $R(x) = \dfrac{P(x)}{Q(x)}$ be a rational function

i.e. a function created from a ratio (quotient) of two polynomials $P(x), Q(x)$.

$R(x)$ has a vertical asymptote at $x=a$ if and only if [iff] $Q(x)$ has more factors of $(x-a)$ than $P(x)$. [in other words, the multiplicity of $(x-a)$ in $Q(x)$ is greater]

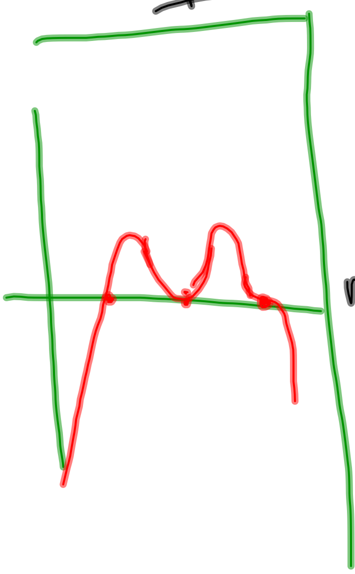
$$R(x) = \frac{(x-1)(x-2)^2(x-3)}{(x-1)(x-2)(x-3)^2} \approx \frac{x-2}{x-3}$$

vert. asymp. $\Longleftrightarrow$ denom $=0$

$x-3=0$

$\boxed{x=3}$

multiplicity.



$y = (x-1)(x-2)^2(x-3)^3$

degree $= 6$

zeros $\Rightarrow$ $x=1, x=2, x=3$

multiplicity of the factors

"mini degree"

| factor | multiplicity | multiplicity of zeros |
|---|---|---|
| $(x-1)$ | 1 | $x=1$ |
| $(x-2)$ | 2 | $x=2$ |
| $(x-3)$ | 3 | $x=3$ |

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Let $R(x) = \frac{P(x)}{Q(x)}$ be a rational function

i.e. a function created from a ratio (quotient) of two polynomials $P(x), Q(x)$.

$R(x)$ has a horizontal asymptote at $y=a$, iff:

1) the degrees of $P(x)$ and $Q(x)$ are the same

look at high degree terms & divide ⇐ [[ $y =$ divide $P(x)$ by $Q(x)$ and ignore remainder ]]

2) the degree of $Q(x) >$ degree of $P(x)$

[ $y=0$ ]

$$y = \frac{(x-1)(x-2)^2(x-3)}{(x-1)(x-2)(x-3)^2(x-4)}$$

2014-09-03 day 6

**1.** For the function $f$ graphed in the accompanying figure, find

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$ (c) $\lim_{x \to 3} f(x)$

(d) $f(3)$ (e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to +\infty} f(x)$.

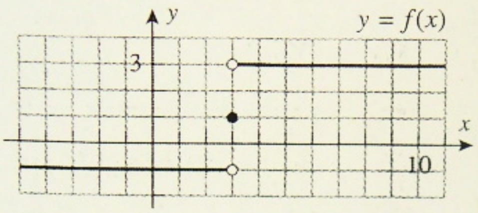


Figure Ex-1

a) $-1$

b) $3$

c) $\lim_{x \to 3} f(x) =$ DNE

d) $1$

e) $\lim_{x \to -\infty} f(x) = -1$

f) $\lim_{x \to \infty} f(x) = 3$

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**3.** For the function $g$ graphed in the accompanying figure, find

(a) $\lim_{x \to 4^-} g(x)$ (b) $\lim_{x \to 4^+} g(x)$ (c) $\lim_{x \to 4} g(x)$

(d) $g(4)$ (e) $\lim_{x \to -\infty} g(x)$ (f) $\lim_{x \to +\infty} g(x)$.

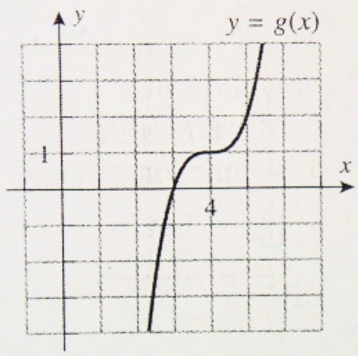


Figure Ex-3

4. For the function g graphed in the accompanying figure, find

a) $\lim_{x \to 4^-} g(x) = 1$

b) $\lim_{x \to 4^+} g(x) = 1$

c) $\lim_{x \to 4} g(x) = 1$

d) $g(4) = 1$

e) $\lim_{x \to -\infty} g(x) =$ DNE but $= -\infty$

f) $\lim_{x \to +\infty} g(x) =$ ~~DNE~~ $+\infty$

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Figure Ex-1

**2.** For the function $f$ graphed in the accompanying figure, find

(a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2} f(x)$

(d) $f(2)$ (e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to +\infty} f(x)$.

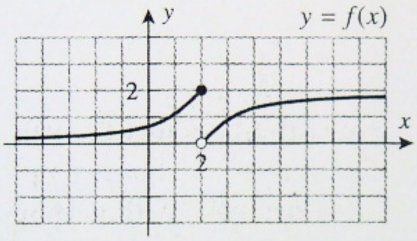


Figure Ex-2

a)$^{lh}$ 2

b)$^{rh}$ 0

c)$^{2s}$ dne

d) $f(2) = 2$

e) $\lim_{\to -\infty} = 0$

f) $\lim_{\to +\infty} = 2$

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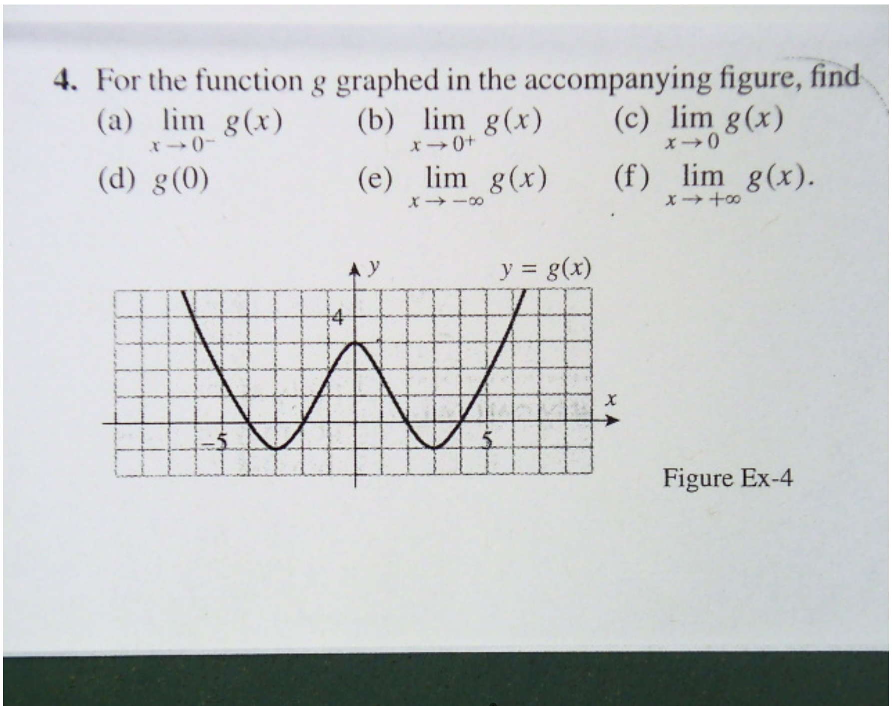

**4.** For the function $g$ graphed in the accompanying figure, find

(a) $\lim_{x \to 0^-} g(x)$ (b) $\lim_{x \to 0^+} g(x)$ (c) $\lim_{x \to 0} g(x)$

(d) $g(0)$ (e) $\lim_{x \to -\infty} g(x)$ (f) $\lim_{x \to +\infty} g(x)$.



Figure Ex-4

a) 3

b) 3

c) 3

d) 3

e) $= +\infty$ f) $= +\infty$